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# Mode Change Protocols for Priority-Driven Preemptive Scheduling



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Real-Time Scheduling in Ada Project

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# Mode Change Protocols for Priority-Driven Preemptive Scheduling

Abstract: In many real-time applications, the set of tasks in the system as well as the characteristics of the tasks change during system execution. Specifically, the system moves from one mode of execution to another as its mission progresses. A mode change is characterized by the deletion of some tasks, addition of new tasks, or changes in the parameters of certain tasks, e.g., increasing the sampling rate to obtain a more accurate result. This paper discusses a protocol for systematically accomplishing mode change in the context of a priority-driven preemptive scheduling environment.

#### 1. Introduction

To successfully develop a large-scale real-time system, we must be able to manage both the logical complexity and timing complexity by using a disciplined approach. The logical complexity is addressed by software engineering methodology, while the timing complexity is addressed by research in real-time scheduling algorithms [2, 3, 5, 6, 7, 8, 10, 12, 14]. An important class of scheduling algorithms is known as static priority scheduling algorithms. These algorithms have several attractive properties. First, they are simple to implement. Second, they have good performance. The utilization bound of a randomly chosen periodic task set is 88% [4], while the worst-case bound for any task set is 69% [7]. In many applications, periodic tasks are often harmonic or nearly harmonic, and this leads to utilization bounds at or near 100%.

In addition to good performance found in practice, static priority scheduling algorithms are analyzable for a wide variety of practical conditions such as the scheduling of a mixture of periodic and aperiodic tasks [3], the handling of transient overloads [11], and the effect of using semaphores [12] and Ada rendezvous [1] for synchronization. From a software engineering point of view, these algorithms translate complex timing constraints into simple resource utilization constraints. As long as the utilization constraints of the CPU, I/O channels and communication media are observed, the deadlines of periodic tasks and the response time requirements of aperiodic tasks will both be met [2]. This means that the real-time software can be modified freely as long as the utilization bounds are observed. Furthermore, should there be a transient overload, the tasks that will miss deadlines will miss them in reverse order of importance, and the number of tasks missing their deadlines will be a function of the overload [11].

However, in many applications neither the task set nor the task parameters can remain static throughout the mission. A change in operational mode often leads to the modification of task parameters (e.g., task period and execution time) as well as the addition of some

new tasks and deletion of some existing tasks. For example, a phase array radar can adjust its sampling rate for the tracking task. Generally speaking, there are two types of mode change issues: application issues and runtime management issues. Application issues deal with the semantics of mode change: the condition for initiating a mode change, the set of tasks to be replaced or modified, and the sequence in which to carry out the mode change activities. In this paper, we do not address the application issues of mode change. We assume that when a mode change is initiated, we are given a list of tasks to be modified, added or deleted, and the sequence to do so.

The focus of this paper is on the runtime management of the mode change process. Specifically, we focus upon the scheduling of mode change activities and of tasks during the transition period of mode change. Our objective is to accomplish the mode change process quickly subject to keeping the consistency of shared data and to meeting the deadlines of tasks that must execute before, during, and after a mode change. This paper is organized as follows. In Section 2, we first review the rate-monotonic algorithm and the priority ceiling protocol for scheduling periodic tasks since our mode change protocol will be designed to be compatible with them. In Section 3, we develop the basic mode change protocol and analyze the properties of the basic protocol. In Section 4, we first examine some possible alternatives to the basic protocol. Next, we consider the interplay between this basic protocol and other scheduling issues, namely, the period transformation method for maintaining stability under transient overload and the server algorithms for scheduling both periodics and aperiodics. Finally in Section 5, we present the concluding remarks.

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# 2. Scheduling Periodic Tasks

In this section, we first review the rate-monotonic scheduling algorithm for independent periodic tasks and then review the priority ceiling protocol designed for the synchronization of periodic tasks using the rate-monotonic scheduling approach. We shall first define the basic concepts and state our assumptions before presenting a review of the scheduling algorithms. A job J with execution time C is a sequence of instructions that will continuously use the processor until its completion if it is executing alone on the processor. That is, we assume that a job J does not suspend itself, say for I/O operations; however, such a situation can be accommodated by defining two or more jobs. In addition, we assume that the critical section of a job is *properly* nested and a job will release all of its locks, if it holds any, before or at the end of its execution. In all our discussions below, we assume that jobs  $J_1, J_2, \cdots$ ,  $J_n$  are listed in descending order of priority with  $J_1$  having the highest priority. A periodic  $\cdots$ }, where T is the period of task  $\tau$ . An aperiodic task is a sequence of the same type of job occurring at irregular intervals. Each task is assigned a fixed priority P, and every job of the same task is initially assigned that task's priority. If several jobs are eligible to run, the highest priority job will be run. Jobs with the same priority are executed in a first-come first-serve discipline. When a job J is forced to wait for the execution of lower priority jobs, job J is said to be "blocked." When a job waits for the execution of higher priority jobs or equal priority jobs that have arrived earlier, it is not considered as blocked.

## 2.1. Scheduling Independent Periodic Tasks

Tasks are considered as independent if they do not synchronize their executions with each other. Given a set of independent periodic tasks, the rate-monotonic scheduling algorithm gives a fixed priority to each task and assigns higher priorities to tasks with shorter periods. A task set is said to be "schedulable" if all its deadlines are met, i.e., if every periodic task finishes its execution before the end of its period. Any set of independent periodic tasks is schedulable by the rate-monotonic algorithm if the condition of Theorem 1 is met [7].

**Theorem 1:** A set of n independent periodic tasks scheduled by the rate-monotonic algorithm will always meet its deadlines, for all task phasings, if

$$\frac{C_1}{T_1} + \cdots + \frac{C_n}{T_n} \le n(2^{1/n}-1)$$

where  $C_i$  and  $T_i$  are the execution time and period of task  $\tau_i$  respectively.

Theorem 1 offers a sufficient (worst-case) condition that characterizes the schedulability of the rate-monotonic algorithm. This bound converges to 69% (*In* 2) as the number of tasks approaches infinity. Table 2-1 shows values of the bound for 1 to 10 tasks.

The utilization bound of Theorem 1 is very pessimistic because the worst-case task set is contrived and rather unlikely to be encountered in practice. For a randomly chosen task set,

Scheduling Bounds					
# of Tasks	Utilization Bound				
1	1.0				
2	0.828				
3	0.779				
4	0.756				
5	0.743				
6	0.734				
7	0.728				
8	0.724				
9	0.720				
10	0.718				

Table 2-1: Worst-Case Scheduling Bounds as a Function of Number of Tasks

bound of Theorem 1 can meet its deadlines, the conditions of Theorem 2 can be checked [4].

**Theorem 2:** A set of n independent periodic tasks scheduled by the rate-monotonic algorithm will always meet its deadlines, for all task phasings, if and only if

$$\forall i, 1 \le i \le n,$$
  $\min \sum_{j=1}^{i} C_j \frac{1}{|T_k|} \frac{|T_k|}{|T_j|} \le 1$ 

$$(k, l) \in R_i$$

where  $C_j$  and  $T_j$  are the execution time and period of task  $\tau_j$  respectively and  $R_j = \{ (k, l) \mid 1 \le k \le i, l = 1, \cdots, \lfloor T_l / T_k \rfloor \}$ .

Theorem 2 provides the exact criterion for testing the schedulability of independent periodic tasks using the rate-monotonic algorithm. In effect, the theorem checks if each task can complete its execution before its first deadline by checking all the scheduling points.<sup>1</sup> The scheduling points for task  $\tau$  are  $\tau$ 's first deadline and the end of periods of higher priority tasks within  $\tau$ 's first deadline. In each application of the formula, i corresponds to the task  $\tau_i$  whose deadline is to be checked, and k corresponds to each of the tasks that affects the completion time of task  $\tau_i$ , i.e., task  $\tau_i$  itself and the higher priority tasks. For given i and k, i represents the scheduling points of task  $\tau_k$ . For example, suppose that we have tasks  $\tau_i$  and  $\tau_i$  with periods  $t_i$  = 5 and  $t_i$  = 14. For task ( $t_i$ ,  $t_i$  = 1) we have only one scheduling point, the end of task  $t_i$ 's first period, i.e.,  $t_i$  = 1 and  $t_i$  = 1 and  $t_i$  = 17.

<sup>&</sup>lt;sup>1</sup>It was shown in [7] that when all the tasks are initiated at the same time, if the first job of a task meets its deadline, that task will never miss a deadline.

The scheduling point is, of course,  $\tau_1$ 's first deadline ( $IT_k = 5$ , I = 1, k = 1). For task ( $\tau_i$ , i = 2), there are *two* scheduling points from all higher priority tasks, ( $\tau_k$ , k = 1), i.e., (I = 1, ...,  $\lfloor T_i/T_k \rfloor = \lfloor T_2/T_1 \rfloor = 2$ ). The two scheduling points correspond to the two end-points of task  $\tau_1$ 's period within the first deadline of task  $\tau_2$  at 14, i.e., ( $IT_k = 5$ , I = 1, K = 1) and ( $IT_k = 10$ , I = 2, K = 1). Finally, there is the scheduling point from  $\tau_2$ 's own first deadline, i.e., ( $IT_k = 14$ , I = 1, I = 1, I = 1). At each scheduling point, we check if the task in question can complete its execution at or before the scheduling point. A detailed illustration of the application of this theorem and its generalization is given in Example 3 in Section 2.3.

# 2.2. Task Synchronization

In the previous sections we have discussed the scheduling of independent tasks. Tasks, however, do interact. In this section, we will discuss how the rate-monotonic scheduling theory can be applied to real-time tasks that must interact. The discussion is limited in this paper to scheduling within a uniprocessor. Readers who are interested in the multiprocessor synchronization problem are referred to [9].

Common synchronization primitives include semaphores, locks, monitors, and Ada rendezvous. Although the use of these or equivalent methods is necessary to protect the consistency of shared data or to guarantee the proper use of non-preemptable resources, their use may jeopardize the ability of the system to meet its timing requirements. In fact, a direct application of these synchronization mechanisms may lead to an indefinite period of priority inversion and low schedulability.

Example 1: Suppose  $J_1$ ,  $J_2$ , and  $J_3$  are three jobs arranged in descending order of priority with  $J_1$  having the highest priority. Let jobs  $J_1$  and  $J_3$  share a data structure guarded by a binary semaphore S. Suppose that at time  $t_1$ , job  $J_3$  locks the semaphore S and executes its critical section. During the execution of job  $J_3$ 's critical section, the high priority job  $J_1$  is initiated, preempts  $J_3$  and later attempts to use the shared data. However, job  $J_1$  will be blocked on the semaphore S. One might expect that  $J_1$ , being the highest priority job, is blocked no longer than the time for job  $J_3$  to complete its critical section. However, the duration of blocking is, in fact, unpredictable. This is because job  $J_3$  can be preempted by the intermediate priority job  $J_2$ . The blocking of  $J_3$ , and hence that of  $J_1$ , will continue until  $J_2$  and any other pending intermediate jobs are completed.

The blocking period in this example can be arbitrarily long. This situation can be partially remedied if a job in its critical section is not allowed to be preempted; however, this solution is only appropriate for very short critical sections, because it creates unnecessary blocking. For instance, once a low priority job enters a long critical section, a high priority job which does not access the shared data structure may be needlessly blocked.

The *priority ceiling protocol* is a real-time synchronization protocol that ensures (1) freedom from mutual deadlock and (2) that a high priority task will be blocked by lower priority tasks for the duration of at most one critical section [1, 12]. Two ideas underlie the design of this

protocol. First is the concept of priority inheritance: when a task  $\tau$  blocks the execution of higher priority tasks, task  $\tau$  should execute at the highest priority level of all the tasks blocked by  $\tau$ . Secondly, we must guarantee that each newly started critical section executes at a priority level that is higher than the (inherited) priority levels of the preempted critical sections. It was shown in [12] that such a prioritized total ordering in the execution of critical sections leads to the two desired properties. To achieve such prioritized total ordering, we define the concept of the priority ceiling of a binary semaphore S to be equal to the highest priority task that may lock S. When a job J attempts to execute one of its critical sections, it will be blocked unless its priority is strictly higher than all the priority ceilings of semaphores currently locked by jobs other than J. If job J blocks, the job that holds the lock on the highest priority ceiling semaphore is said to be blocking J and hence inherits Js priority. A job J can, however, always preempt another job executing at a lower priority level as long as J does not attempt to enter a critical section.

Example 2: Suppose that we have two jobs  $J_1$  and  $J_2$  in the system. In addition, there are two shared data structures protected by binary semaphores  $S_1$  and  $S_2$  respectively. Suppose the sequence of processing steps for each job is as follows.

$$J_1 = \{ \cdots, P(S_1), \cdots, P(S_2), \cdots, V(S_2), \cdots, V(S_1), \cdots \}$$

$$J_2 = \{ \cdots, P(S_2), \cdots, P(S_1), \cdots, V(S_1), \cdots, V(S_2), \cdots \}$$

Recall that the priority of job  $J_1$  is assumed to be higher than that of job  $J_2$ . Thus, the priority ceilings of both semaphores  $S_1$  and  $S_2$  are equal to the priority of job  $J_1$ . Suppose that at time  $t_0$ ,  $J_2$  is initiated and it begins execution and then locks semaphore  $S_2$ . At time  $t_1$ , job  $J_1$  is initiated and preempts job  $J_2$  and at time  $t_2$ , job  $J_1$  tries to enter its critical section by making an indivisible system call to execute  $\mathbf{P}(S_1)$ . However, the runtime system will find that job  $J_1$ 's priority is *not* higher than the priority ceiling of *locked* semaphore  $S_2$ . Hence, the runtime system suspends job  $J_1$  without locking  $S_1$ . Job  $J_2$  now *inherits* the priority of job  $J_1$  and resumes execution. Note that  $J_1$  is blocked outside its critical section. As  $J_1$  is not given the lock on  $S_1$  but suspended instead, the potential deadlock involving  $J_1$  and  $J_2$  is prevented. Once  $J_2$  exits its critical section, it will return to its assigned priority and immediately be preempted by job  $J_1$ . Then,  $J_1$  will execute to completion, and finally  $J_2$  will resume and run to completion.

Let  $B_i$  be the longest duration of blocking that can be experienced by a job of task  $\tau_i$ . The following two theorems indicate whether the deadlines of a set of periodic tasks can be met if the priority ceiling protocol is used.

**Theorem 3:** A set of *n* periodic tasks using the priority ceiling protocol can be scheduled by the rate-monotonic algorithm if the following condition is satisfied [12]:

$$\frac{C_1}{T_1} + \cdots + \frac{C_n}{T_n} + \max \left( \frac{B_1}{T_1}, \cdots, \frac{B_{n-1}}{T_{n-1}} \right) \le n(2^{1/n}-1)$$

**Theorem 4:** A set of *n* periodic tasks using the priority ceiling protocol can be

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scheduled by the rate-monotonic algorithm for all task phasings if the following condition is satisfied [12].

$$\forall i, 1 \le i \le n, \qquad \min \left( \sum_{j=1}^{i-1} C_j \frac{1}{|T_k|} \left[ \frac{|T_k|}{|T_j|} \right] + \frac{C_i}{|T_k|} + \frac{B_i}{|T_k|} \right) \le 1$$

$$(k, l) \in R_i$$

where  $C_i$ ,  $T_i$  and  $R_i$  are defined in Theorem 2, and  $B_i$  is the worst-case blocking time for a job of task  $\tau_i$ .

<u>Remark:</u> Theorems 3 and 4 generalize Theorems 1 and 2 by taking the blocking duration of a job into consideration. The  $B_i$ 's in Theorems 3 and 4 can be used to account for any delay caused by resource sharing. Note that the upper limit of the summation in the theorem is (i-1) instead of i, as in Theorem 2.

In the application of Theorems 3 and 4, it is important to realize that under the priority ceiling protocol, a task  $\tau$  can be blocked by a lower priority task  $\tau_L$  if  $\tau_L$  may lock a semaphore S whose priority ceiling is higher than or equal to the priority of task  $\tau$ , even if  $\tau$  and  $\tau_L$  do not share any semaphore. For example, suppose that  $\tau_L$  locks S first. Next,  $\tau$  is initiated and preempts  $\tau_L$ . Later, a high priority task  $\tau_H$  is initiated and attempts to lock S. Task  $\tau_H$  will be blocked. Task  $\tau_L$  now inherits the priority of  $\tau_H$  and executes. Note that  $\tau$  has to wait for the critical section of  $\tau_L$  even  $\tau$  and  $\tau_L$  do not share any semaphore. We call such blocking, "push-through blocking". Push-through blocking is the price for avoiding unbounded priority inversion. If task  $\tau_L$  does not inherit the priority of  $\tau_H$ , task  $\tau_H$  can be indirectly preempted by task  $\tau$  and all the tasks that have priority higher than that of  $\tau_L$ . Finally, we want to point out that even if task  $\tau_H$  does not attempt to lock S but attempts to lock another unlocked semaphore,  $\tau_H$  will still be blocked by the priority ceiling protocol because  $\tau_H$ 's priority is not higher than the priority ceiling of S. We call this form of blocking, "ceiling blocking". Ceiling blocked at most once.

## 2.3. An Example

In this section, we give a simple example to illustrate the application of the scheduling theorems.

Example 3: Consider the following task set.

- 1. Periodic task  $\tau_1$ : execution time = 40 msec; period = 100 msec; deadline is at the end of each period.
  - In addition,  $\tau_3$  may block  $\tau_1$  for 10 msec through the use of a shared communication server and task  $\tau_2$  may block  $\tau_1$  for 20 msec through the use of a shared data object.
- 2. Periodic task  $\tau_2$ : execution time = 40 msec; period = 150 msec; deadline is 20 msec before the end of each period.
- 3. Periodic task  $\tau_3$ : execution time = 100 msec; period = 350 msec; deadline is at the end of each period.

<u>Solution</u>: Since under the priority ceiling protocol a task can be blocked by lower priority tasks at most once, the maximal blocking time for task  $\tau_1$  is  $B_1 = \max(10, 20)$  msec = 20 msec. Since  $\tau_3$  may lock the semaphore  $S_c$  associated with the communication server and the priority ceiling of  $S_c$  is higher than that of task  $\tau_2$ , task  $\tau_2$  can be blocked by task  $\tau_3$  for 10 msec.<sup>2</sup> Finally, task  $\tau_2$  has to finish 20 msec earlier than the nominal deadline of a periodic task. This is equivalent to saying that  $\tau_2$  will always be blocked for additional 20 msec but its deadline is at the end of the period. Hence,  $B_2 = (10 + 20)$  msec = 30 msec.<sup>3</sup> Using Theorem 4:

- 1. Task  $\tau_1$ : Check  $C_1 + B_1 \le 100$ . Since  $40 + 20 \le 100$ , task  $\tau_1$  is schedulable.
- 2. Task  $\tau_2$ : Check whether either

$$C_1 + C_2 + B_2 \le 100$$
 80 + 30 > 100  
or  $2C_1 + C_2 + B_2 \le 150$  120 + 30 = 150

Task  $\tau_2$  is schedulable and in the worst-case phasing will meet its deadline exactly at time 150.

3. Task  $\tau_3$ : Check whether either

$$C_1 + C_2 + C_3 \le 100 \qquad \qquad 40 + 40 + 100 > 100$$
 or 
$$2C_1 + C_2 + C_3 \le 150 \qquad \qquad 80 + 40 + 100 > 150$$
 or 
$$2C_1 + 2C_2 + C_3 \le 200 \qquad \qquad 80 + 80 + 100 > 200$$
 or 
$$3C_1 + 2C_2 + C_3 \le 300 \qquad \qquad 120 + 80 + 100 = 300$$
 or 
$$4C_1 + 3C_2 + C_3 \le 350 \qquad \qquad 160 + 120 + 100 > 350$$

Task  $\tau_3$  is also schedulable and in the worst-case phasing will meet its deadline exactly at time 300. It follows that all the three periodic tasks can meet their deadlines.

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<sup>&</sup>lt;sup>2</sup>This may occur if  $\tau_3$  blocks  $\tau_1$  and inherits  $\tau_1$ 's priority.

<sup>&</sup>lt;sup>3</sup>Note that the blocked-at-most-once result does not apply here. It only applies to blocking caused by task synchronization using the priority ceiling protocol.

# 3. Mode Change Protocols

We now discuss the protocols needed to support mode changes in the context of our scheduling algorithms for periodic tasks. First, we discuss the characteristics of mode change. This is followed by a simple protocol when only independent tasks are involved. Finally, we discuss the mode change problems in the presence of task interactions.

# 3.1. Mode Changes for Independent Tasks

From a scheduling point of view, typical mode change operations can be classified into two types:

- 1. Operations that increase a task set's processor utilization:
  - a. Adding a task
  - b. Increasing the execution time of a task
  - c. Increasing the frequency of execution of a task.
- 2. Operations that decrease a task set's processor utilization:
  - a. Deleting a task
  - b. Decreasing the execution time of a task
  - c. Decreasing the frequency of a task.

A simple mode change protocol can be defined in terms of the deletion of existing tasks and the addition of a new task. If a task modifies its parameters, e.g., changes its sampling rate, it is modeled as the deletion of the original task and the addition of a new task. In addition, we assume that all the tasks are periodic and that a task which has started its execution will not be deleted until it has completed its execution in the current period. These assumptions will be relaxed later in this paper.

When tasks are independent, the addition, deletion, or modification of a task's parameters is merely an application of Theorems 1 or 2.

**Theorem 5:** At any time t, a task  $\tau$  can be added, or its computation time C increased or its frequency increased without causing any task to miss their deadlines if the conditions of Theorems 1 or 2 are satisfied.

**Proof:** It directly follows the fact that a task set is schedulable if it satisfies the conditions of Theorems 1 or 2.

**Theorem 6:** At any time t, a task  $\tau$  can be deleted, or its computation time C reduced or its frequency reduced without causing any task to miss their deadlines.

**Proof:** It directly follows the fact that if a given task set satisfies the conditions of Theorems 1 or 2, then the modified task set will also satisfy the conditions in question.

It may seem that once a task is deleted, its allocated processor capacity can be immediately

reused by other tasks. However, this is not true. The schedulability of a set of tasks using the rate-monotonic algorithm is determined under the assumption that once a job J of a task  $\tau$  is initiated, task  $\tau$  cannot request additional processing until the beginning of  $\tau$ 's next period. Thus, even if job J has finished its execution m units before the end of  $\tau$ 's current period, task  $\tau$  has used up the processor capacity for the given period. Hence, task  $\tau$  must be included in the application of Theorems 1 and 2 3 and 4 until the end of the current period. In other words, the processor capacity allocated to  $\tau$  cannot be used by new tasks until the end of  $\tau$ 's current period.

## 3.2. The Basic Mode Change Protocol

In this section, we will develop a basic mode change protocol for periodic tasks using binary semaphore for synchronization. There are two basic concepts in the design of this protocol. The first is the notion of sufficient processor capacity to add a task "on the fly" when synchronization is involved. The second is the preservation of the characteristic of the priority ceiling protocol: each newly started critical section is guaranteed to execute at a priority level that is higher than the maximum priority that any of the preempted critical sections can inherit.

<u>Definition</u>: Processor capacity is said to be sufficient for adding a task  $\tau$ , if the resulting n tasks, including  $\tau$ , can meet all their deadlines using the rate-monotonic algorithm and the priority ceiling protocol.

Theorems 3 and 4 provide us with sufficient conditions for processor capacity to be sufficient. Theorem 4 allows for a higher degree of processor utilization while Theorem 3 is easier to apply.

We have defined the concept of having sufficient capacity to add a task. A related concept is the deletion of a task  $\tau$  and reclaiming the processor capacity used by  $\tau$ .

<u>Definition</u>: The processor capacity used by a deleted task  $\tau$  is said to be reclaimed at time t if after t task  $\tau$  does not need to be included in the application of Theorems 3 and 4.

We now define the basic mode change protocol. We assume that during mode transition, tasks are deleted/added in an order that is consistent with the semantics of the application.

- The addition and/or the deletion of tasks in mode change may lead to the modification of the priority ceilings of some semaphores across the mode change. Upon the initiation of mode change,
  - For each of the unlocked semaphores S, whose priority ceiling needs to be raised, S's ceiling is raised immediately and indivisibly.
  - For each locked semaphore S, whose priority ceiling needs to be raised, S's priority ceiling is raised immediately and indivisibly after S is unlocked.

- For each semaphore S, whose priority ceiling needs to be lowered, S's priority ceiling is lowered when all the tasks which may lock S and which have priorities greater than the new priority ceiling of S are deleted.
- 2. A task  $\tau$ , which needs to be deleted, can be deleted immediately upon the initiation of mode change, if  $\tau$  has not yet started its execution in its current period. In addition, the processor capacity used by  $\tau$  is reclaimed immediately. On the other hand, if  $\tau$  has started execution,  $\tau$  can be deleted after the end of its execution and before its next initiation time. The processor capacity allocated to  $\tau$  will, however, not be reclaimed until the next initiation time.
- 3. A task  $\tau$  can be added into the system if the following two conditions are met:
  - If task  $\tau$ 's priority is higher than the priority ceilings of locked semaphores  $S_1$ , ...,  $S_k$ , then the priority ceilings of  $S_1$ , ...  $S_k$  must be first raised before adding task  $\tau$ .
  - There must be sufficient processor capacity for adding task  $\tau$ .

We now illustrate the mode change protocol using an example.

Example 4: Suppose that the task set  $\{\tau_1, \tau_2, \tau_3\}$  is replaced by the task set  $\{\tau_0, \tau_3^*, \tau_4, \tau_5\}$ . In other words, tasks  $\tau_1$  and  $\tau_2$  are to be deleted and replaced with  $\tau_0$ ,  $\tau_4$  and  $\tau_5$  in the new task set.  $\tau_3$  is to be modified to  $\tau_3^*$  resulting in a change of parameters. Suppose that  $\tau_0$  cannot be added until  $\tau_1$  is deleted because of insufficient processor capacity or semantic requirements. Similarly, suppose that  $\tau_4$  and  $\tau_5$  cannot be added until  $\tau_2$  is deleted and its processor capacity reclaimed. We assume that we add tasks  $\tau_0$ ,  $\tau_4$  and  $\tau_5$  in that order when a mode change is initiated. In addition, we assume that tasks that need to be deleted can be deleted in any order.

Let the jobs of each task execute the following sequences of instructions in the current task set.

$$J_{1} = \{ \cdots, P(S_{1}), \cdots, V(S_{1}), \cdots \}$$

$$J_{2} = \{ \cdots, P(S_{1}), \cdots, P(S_{2}), \cdots, V(S_{2}), \cdots, V(S_{1}), \cdots \}$$

$$J_{3} = \{ \cdots, P(S_{2}), \cdots, V(S_{2}), \cdots \}$$

Let the jobs in the new mode execute the following sequences of events:

$$J_{0} = \{ \cdots, P(S_{2}), \cdots, V(S_{2}), \cdots \}$$

$$J_{3} = \{ \cdots, P(S_{2}), \cdots, V(S_{2}), \cdots \}$$

$$J_{4} = \{ \cdots, P(S_{2}), \cdots, P(S_{1}), \cdots, V(S_{1}), \cdots, V(S_{2}), \cdots \}$$

$$J_{5} = \{ \cdots, P(S_{1}), \cdots, V(S_{1}), \cdots \}$$

As before, we assume that the priority of  $J_{i+1}$  is lower than the priority of  $J_i$ . Before the mode

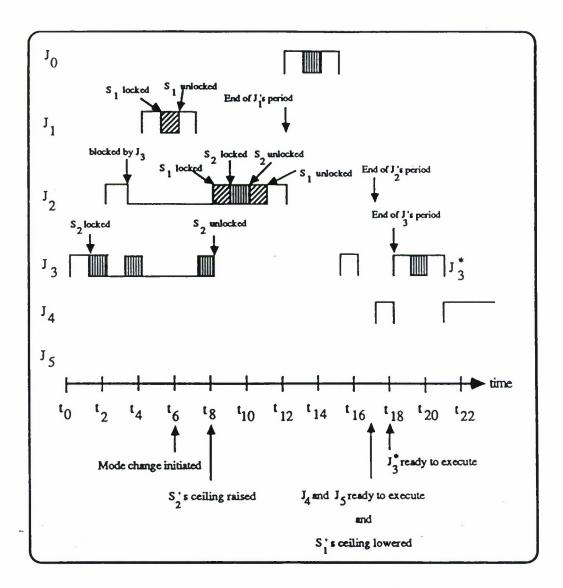


Figure 3-1: Sequence of Events Described in Example 4.

change, the priority ceilings of  $S_1$  and  $S_2$  are the priorities of  $\tau_1$  and  $\tau_2$  respectively. However, after the mode change, the priority ceilings of  $S_1$  and  $S_2$  are the priorities of  $\tau_4$  and  $\tau_0$  respectively. Thus, after the mode change, the priority ceiling of  $S_1$  is lowered, while that of  $S_2$  is raised.

Consider the following sequence of events depicted in Figure 3-1. A line at a low level indicates that the corresponding job is blocked or has been preempted by a higher priority job. A line raised to a higher level indicates that the job is executing. The absence of a line indicates that the job has not yet been initiated or has completed. Shaded portions indicate execution of critical sections.

- At time  $t_0$ , the task set that is being run is  $\{\tau_1, \tau_2, \tau_3\}$ .  $J_3$  arrives and begins execution.
- At time  $t_1$ ,  $J_3$  locks  $S_2$  and enters its critical section.
- At time  $t_2$ ,  $J_2$  arrives and preempts  $J_3$ .
- At time  $t_3$ ,  $J_2$  attempts to lock  $S_1$  and is blocked by the priority ceiling protocol.  $J_3$  inherits  $J_2$ 's priority and resumes execution.
- At time  $t_4$ ,  $J_1$  arrives and preempts  $J_3$ .
- At time  $t_5$ ,  $J_1$  successfully locks  $S_1$ , since its priority is higher than the priority ceiling of locked semaphore  $S_2$ .
- At time t<sub>6</sub>, J<sub>1</sub> releases the semaphore S<sub>1</sub>. At the same time, a mode change is initiated due to external requirements. τ<sub>0</sub> is the first task to be added at the mode change and it cannot be added until the processing capacity is reclaimed from τ<sub>1</sub>. Hence, τ<sub>0</sub> cannot be added until the end of J<sub>1</sub>'s current period (at t<sub>12</sub>). Similarly, τ<sub>4</sub> and τ<sub>5</sub> cannot be added until the end of J<sub>2</sub>'s current period (at t<sub>17</sub>). The priority ceiling of S<sub>2</sub> gets raised in the new mode but cannot be raised until it is unlocked.
- At time  $t_7$ ,  $J_1$  completes execution and  $J_3$  resumes execution at its inherited priority of  $J_2$ .
- At time  $t_8$ ,  $J_3$  releases the semaphore  $S_2$  and resumes its original priority. The priority ceiling of  $S_2$  is raised now.  $J_2$  immediately preempts  $J_3$  and locks  $S_1$ .
- At time  $t_9$ ,  $J_2$  makes a nested access to  $S_2$  and locks  $S_2$ .
- At time  $t_{10}$ ,  $J_2$  releases the semaphore  $S_2$ .
- At time  $t_{11}$ ,  $J_2$  releases the semaphore  $S_1$ .
- At time  $t_{12}$ ,  $J_2$  completes execution. The current period of  $\tau_1$  ends here and  $\tau_1$  is deleted. Hence,  $\tau_0$  is added into the system and immediately becomes eligible for execution.
- At time  $t_{13}$ ,  $J_0$  locks semaphore  $S_2$  since there is no other locked semaphore in the system.
- At time  $t_{14}$ ,  $J_0$  releases  $S_2$ .
- ullet At time  $t_{15}$ ,  $J_0$  completes execution and  $J_3$  resumes execution.
- At time  $t_{16}$ ,  $J_3$  completes execution.
- The processor remains idle during the interval  $[t_{16}, t_{17}]^4$ .
- At time  $t_{17}$ , the current period of  $\tau_2$  ends and it can be deleted from the system. The priority ceiling of  $S_1$  is lowered and  $\tau_0$  is added into the system. Now,  $\tau_4$  and  $\tau_5$  can also be added into the system. Having the highest priority among tasks ready to run,  $J_4$  begins execution.
- At time  $t_{18}$ ,  $\tau_3$ 's current period ends and  $\tau_3$  can be replaced with  $\tau_3^*$ . The mode change is now complete. Job  $J_3$  preempts  $J_4$  and begins execution.

<sup>&</sup>lt;sup>4</sup>Idling of the processor can occur for two reasons: the rate-monotonic algorithm does not guarantee a 100% schedulability level for all task sets. Secondly, task sets in some modes may have lower processor utilization levels than task sets in other modes.

- At time  $t_{21}$ ,  $J_3$  completes execution, locking and releasing  $S_2$  at  $t_{19}$  and  $t_{20}$  respectively. Now,  $J_A$  resumes execution.
- Processing proceeds normally in the new mode.

The above example illustrates the following properties of the mode change protocol. First, tasks can be added as long as they are schedulable in the resulting task set. However, a task to be added may have to wait for the deletion of an existing task even though there is idle capacity available. We shall further study this mode change delay in Section 3.4. Task modifications, such as the modification of  $\tau_3$  into  $\tau_3^*$  can be carried out relatively easily.

# 3.3. Properties of The Basic Mode Change Protocol

Under the priority ceiling protocol, there is no mutual deadlock, and a job can be blocked by lower priority jobs for at most the duration of a single critical section [12]. We shall now prove that both these properties are preserved under the mode change protocol.

**Lemma 7:** Under the mode change protocol, when a job J enters its critical section and preempts job  $J_i$  while  $J_i$  is in its critical section, the priority of J is higher than the priority that can be inherited by  $J_i$ .

**Proof:** Under the definition of the mode change protocol, the priority ceiling of a semaphore S will not be lower than the priority of any job that may lock S. Since a job J is allowed to enter its critical section only if Js priority is higher than the priority ceilings of all the semaphores locked by jobs other than J and since the highest priority that a job can inherit is bounded by the priority ceiling of the semaphores locked by this job, it follows that when job J enters its critical section, its priority will be higher than the (inherited) priority of the jobs preempted by J.

Theorem 8: There is no mutual deadlock under the mode change protocol.

**Proof:** Suppose that there is a mutual deadlock. Let the highest priority of all the jobs involved in the deadlock be *P*. Due to the transitivity of priority inheritance, all the jobs involved in the deadlock will eventually inherit the same highest priority *P*. This contradicts Lemma 7.

**Lemma 9:** A job J can be blocked by a lower priority job  $J_{L}$  at most for the duration of executing one critical section.

**Proof:** First, if job  $J_{L}$  is not already in its critical section when job J arrives, then job  $J_{L}$  will be preempted by J and cannot block J. Suppose that  $J_{L}$  is in its critical section when J arrives and that  $J_{L}$  blocks J.  $J_{L}$  inherits the priority of J and continues its execution. Once  $J_{L}$  exits its critical section, by the definition of the priority ceiling protocol,  $J_{L}$  will be assigned its original priority and be immediately preempted by J. Hence,  $J_{L}$  cannot block J again.

**Theorem 10:** Under the mode change protocol, a job J can be blocked by lower priority jobs for at most the duration of a single (outermost) critical section.

**Proof:** Suppose that job J is blocked by lower priority jobs more than once. By Lemma 9, job J must be blocked by n different lower priority jobs,  $J_1, ..., J_n$ , where the priority of  $J_i$  is assumed to be higher than or equal to that of  $J_{i+1}$ . Since a lower priority job cannot block a higher priority job unless it is already in its critical

section, jobs  $J_1$ , ...,  $J_n$  must be in their critical sections when J arrives. By assumption, J is blocked by  $J_n$  and  $J_n$  inherits the priority of J. It follows that job Js priority cannot be higher than the highest priority P that can be inherited by  $J_n$ . On the other hand, by lemma 7, job  $J_{n-1}$ 's priority is higher than P. It follows that job  $J_{n-1}$ 's priority is higher than that of jobs J. This contradicts the assumption that Js priority is higher than that of jobs  $J_1$ , ...,  $J_n$ .

Remark: It is important to point out that the property of a job being blocked for at most one critical section depends upon our model of a job, an instance of a periodic task. We assume that when a job executes alone on the processor, input data from I/O devices will be ready when the job is initiated and it will continue to execute until it completes without suspension for I/O activities. In some applications, an instance of a periodic task may need to suspend itself for I/O. In this case, we have the following corollary:

**Corollary 11:** If a generalized job J suspends itself n times during its execution, it can be blocked for the duration of at most n+1 critical sections.

## 3.4. Mode Change Delays

In this section, we analyze the delays that can occur before a mode change is completed. In the following analysis, we assume that the given task set is schedulable using the ratemonotonic algorithm and the priority ceiling protocol.

<u>Notation</u>: Let  $t_0$  denote the time at which the mode change is initiated. Let  $D_s$  be the delay in elevating semaphore priority ceilings, that is, the delay between  $t_0$  and the time at which all the semaphores whose ceilings need to be raised are raised. Let  $D_c$  be the delay in reclaiming processor capacity, that is, the delay between  $t_0$  and the time at which all the tasks that need to be deleted are deleted and their allocated processor capacity becomes available. Finally, let D be the mode change delay, that is, the duration between  $t_0$  and the time at which the mode change is completed.

The following lemmas and theorems are based on the assumption that the task sets before and after a mode change are schedulable.

Lemma 12: Let  $S_i$  be a semaphore whose priority ceiling needs to be raised across a mode change. Let  $\tau$  be a task whose priority is equal to the priority ceiling of semaphore  $S_i$ . The delay in elevating the priority ceiling of  $S_i$  is bounded by the period of task  $\tau$ , T.

**Proof:** The priority ceiling of a semaphore S can be raised only if it is not locked. Semaphore S may have been locked when the mode change is initiated. However, under the assumption that task  $\tau$  can meet its deadline, the locking of S cannot be longer than T and the lemma follows.

**Lemma 13:** Let S be the semaphore that has the lowest priority ceiling of all the semaphores whose ceilings need to be raised. The ceiling elevation delay for mode change,  $D_s$ , is bounded by T, the period of a task whose priority is equal to the priority ceiling of S.

**Proof:** It directly follows from Lemma 12 and from the fact that a task associated with a lower priority ceiling has a longer period under the rate-monotonic scheduling algorithm.

**Lemma 14:** Let task  $\tau$  be the lowest priority task needed to be deleted. The delay due to the reclamation of processor capacity,  $D_{\rm b}$ , is bounded by the period of task  $\tau$ .

**Proof:** Let the periods of the tasks that need to be deleted be  $\{T_j,...,T_m\}$ , where  $T_k \ge T_j$  if k > j. Under the assumption that the set of given tasks is schedulable, each of the tasks needed to be deleted can be deleted by the end of its current period and its allocated processor capacity can be reclaimed. Hence, we have  $D_c = max\{T_j,...,T_m\}$ . Under the rate-monotonic scheduling algorithm, a task with a longer period has lower priority. It follows that the delay due to reclaiming all the processor capacity is bounded by the period of task  $\tau_m$ ,  $T_m$ .

**Theorem 15:** The mode change delay D is bounded by  $max(D_s, D_c)$ .

**Proof:** Suppose that the mode change request occurs at time  $t_0$ . By  $(t_0 + D_{\rm s})$ , all the semaphore priority ceilings that need to be raised have been raised. By  $(t_0 + D_{\rm c})$ , all the tasks in the current mode that need to be deleted are deleted. That is, the processor capacity that needs for all the new tasks is available by  $(t_0 + D_{\rm c})$ . It follows that all the new tasks can be added by the time  $(t_0 + max(D_{\rm s}, D_{\rm c}))$ . Finally, by the definition of the basic mode change protocol, all the semaphore priority ceilings that need to be lowered have been lowered by  $(t_0 + D_{\rm c})$ . Hence, the mode change delay is bounded by  $max(D_{\rm s}, D_{\rm c})$ .

Remark: In the worst case, the longest period in a task set can be equal to the least common multiple (LCM) of all the periods. By Theorem 15, the mode change delay for such a task set is bounded by this longest period. Thus, the worst-case mode change delay is equal to that of the cyclical executive. In the cyclical executive approach, the major cycle is the LCM of all the periods and a mode change will not be initiated until the current major cycle completes. However, the delay to complete a mode change using the mode change protocol would typically be much shorter than the delay using the cyclical executive approach. In addition, the mode change protocol also provides the flexibility of adding the most urgent task in the new mode first.

# 4. Extensions of the Basic Mode Change Protocol

In this section, we will examine some design alternatives to the mode change protocol as well as the integration of our basic mode change protocol with other scheduling algorithms.

#### 4.1. Variations of the Basic Protocol

The objective in the design of the mode change protocol is to minimize the mode change delay subject to keeping the shared data consistent and to meeting all the deadlines of tasks that must be continuously executing. We also made an implicit assumption that the mode change protocol should not lower the system schedulability in any given mode.

However, assumptions and objectives are, of course, application dependent. Generally, there is relatively little that one can do about mode change delay caused by reclaiming processor capacity, because a task could have started its execution when the mode change is initiated. Once a task begins execution, it may well be desirable to let it complete because the abortion of a task may lead to complications that makes later correction and/or recovery time-consuming. There is an exception to this general observation, however. In certain applications, one can define a set of tasks that constitutes an atomic configuration unit. Such an unit encapsulates all the shared variables for the task set in question. In this case, the application semantic may allow the entire unit to be deleted immediately and indivisibly at the initiation of mode change.

Generally, when  $D_{\rm S} > D_{\rm C}$ , there is an incentive to minimize  $D_{\rm S}$ . We can minimize the mode change delay associated with elevating the priority ceilings if we are willing to pay a schedulability cost. For example, we define the *global ceiling* mode change protocol as follows. In this protocol, the priority ceiling of a semaphore S is defined as the priority of the highest priority task that may access S across all modes. The disadvantage of this mode change protocol is rather obvious. In any mode, the "actual" ceiling of a semaphore can be much lower than the "global" priority ceiling. As a consequence, the blocking duration is longer and it translates into schedulability cost, due to which, some, otherwise schedulable, task sets may become unschedulable. The priority ceiling elevation cost can be fine-tuned, however. Since  $D_{\rm S}$  is determined solely by the period of the task whose priority equals the lowest priority ceiling that needs to be raised,  $D_{\rm S}$  can be shortened by deliberately assigning a higher priority ceiling to the semaphore with this lowest priority ceiling that needs to be raised in mode changes.

Finally, we may want to emphasize the simplicity of managing a mode change process. In this case, we do not raise the semaphore priority ceiling of any semaphore until all the tasks that need to be deleted are deleted and the priority ceilings of associated semaphores are lowered. New tasks will be added at time  $t_{\rm add} = t_0 + (D_s + D_c)$ . We need apply neither Theorem 3 nor Theorem 4 during runtime as long as tasks are known to be schedulable in each mode. This is because at time  $t_{\rm add}$  all the deleted tasks' processor capacity have already been reclaimed and the priority ceilings are at the correct level. That is, the condition under which we may apply Theorem 3 or 4 is the same as in the new mode.

## 4.2. Stability Under Transient Overload

In the previous section, the computation time of a task is assumed to be constant. However, in many applications, task execution times are often stochastic, and the worst-case execution time can be significantly larger than the average execution time. In order to have a reasonably high average processor utilization, we must deal with the problem of transient overload. We consider a scheduling algorithm to be *stable* if there exists a set of *critical* tasks such that all tasks in the set will meet their deadlines even if the processor is overloaded. This means that under worst-case conditions, tasks outside the critical set may miss their deadlines. The rate monotonic algorithm is stable in the sense that the set of tasks that never miss their deadlines does not change as the processor gets more overloaded or as task phasings change. Of course, which tasks are in the critical task set depends on the worst-case utilizations of the particular tasks being considered. The important point is that the rate monotonic theory guarantees that if such a set exists, it always consists of tasks with the highest priorities. This means that if a transient overload should develop, tasks with longer periods will miss their deadlines.

Of course, a task with a longer period could be more critical to an application than a task with a shorter period. One might attempt to ensure that the critical task always meets its deadline by assigning priorities according to a task's importance. However, this approach can lead to poor schedulability, i.e., with this approach, deadlines of critical tasks might be met only when the total utilization is low.

The *period transformation* technique can be used to ensure high utilization while meeting the deadline of an important, long-period task. Period transformation means turning a long-period important task into a high priority task by splitting its work over several short periods. For example, suppose task  $\tau$  with a long period T is not in the critical task set and must never miss its deadline. We can make  $\tau$  simulate a short period task by giving it a period of T/2 and suspending it after it executes half its worst-case execution time, C/2. The task is then resumed and finishes its work in the next execution period. It still completes its total computation before the end of period T. From the viewpoint of the rate monotonic theory, the transformed task has the same utilization but a shorter period, T/2, and its priority is raised accordingly. It is important to note that the most important task need not have the shortest period. We only need to make sure that it is among the first T0 high priority tasks whose worst-case utilization is within the scheduling bound. A systematic procedure for period transformation with minimal task partitioning can be found in [11].

Period transformation allows important tasks to have higher priority while keeping priority assignments consistent with rate-monotonic rules. This kind of transformation should be familiar to users of cyclic executives. The difference here is that we don't need to adjust the code segment sizes so different code segments fit into shared time slots. Instead,  $\tau$  simply requests suspension after performing C/2 amount of work. Alternatively, the runtime scheduler can be instructed to suspend the task after a certain amount of computation has

been done, without affecting the application code.5

The period transformation approach has another benefit—it can raise the rate-monotonic utilization bound. Suppose the rate-monotonic utilization bound is  $U_{max} < 100\%$ , i.e., total task utilization cannot be increased above  $U_{max}$  without missing a deadline. When a period transformation is applied to the task set,  $U_{max}$  will rise. For example:

#### Example 1: Let

- Task  $\tau_1$ :  $C_1 = 4$ ;  $T_1 = 10$ ;  $U_1 = .400$
- Task  $\tau_2$ :  $C_2 = 6$ ;  $T_2 = 14$ ;  $U_1 = .428$

The total utilization is .828, which just equals the bound of Theorem 1, so this set of two tasks is schedulable. If we apply Theorem 2, we find:

$$C_1 + C_2 \le T_1$$
  $4 + 6 = 10$   $l = 1, k = 1$   
or  $2C_1 + C_2 \le T_2$   $8 + 6 = 14$   $l = 1, k = 2$ 

So Theorem 2 says the task set is just schedulable. Now suppose we perform a period transformation on task  $\tau_1$ , so  $C_1' = 2$  and  $T_1' = 5$ . The total utilization is the same and the set is still schedulable, but when we apply Theorem 2 we find:

The second equation shows that the compute times for tasks  $\tau_1$  and/or  $\tau_2$  can be increased without violating the constraint. For example, the compute time of Task  $\tau_1$  can be increased by 2/3 units to 2.667, giving an overall schedulable utilization of 2.667/5 + 6/14 = .961, or the compute time of Task  $\tau_2$  can be increased to 8, giving an overall schedulable utilization of 2/5 + 8/14 = .971. So the effect of the period transformation has been to raise the utilization bound from .828 to at least .961 and at most .971. Indeed, if periods are uniformly harmonic, i.e., if each period is an integral multiple of each shorter period, the utilization bound of the rate-monotonic algorithm is 100%. So the utilization bound produced by the rate monotonic approach is only an upper bound on what can be achieved if the periods are not transformed. Of course, as the periods get shorter, the scheduling overhead utilization increases, so the amount of useful work that can be done decreases. For example, before a period transformation, the utilization for a task, including scheduling overhead, is (C + 2S)/T.

<sup>&</sup>lt;sup>5</sup>The scheduler must ensure that  $\tau$  is not suspended while in a critical region since such a suspension can cause other tasks to miss their deadlines. If the suspension time arrives but the task is in a critical region, then the suspension should be delayed until the task exits the critical region. To account for this effect on the schedulability of the task set, the worst-case execution time must be increased by  $\varepsilon$ , the extra time spent in the critical region, i.e.,  $\tau$ 's utilization becomes  $(0.5C+\varepsilon)/0.5T$ .

<sup>&</sup>lt;sup>6</sup>For example, by transforming the periods in Example 3 so  $\tau'_1$  and  $\tau'_2$  both have periods of 50, the utilization bound is 100%, i.e., 4.7% more work can be done without missing a deadline.

After splitting the period into two parts, the utilization is (.5C + 2S)/.5T, so scheduling overhead is a larger part of the total utilization. However, the utilization bound is also increased, in general. If the increase in utilization caused by the scheduling overhead is less than the increase in the utilization bound, then the period transformation is a win—more useful work can be done while meeting all deadlines.

Period transformation does not affect the mode change protocol except that to delete a transformed task that has already started execution, we must wait for its completion which may take several "transformed periods". In addition, we cannot reclaim the processor capacity of a transformed task until the end of the last transformed period, which is also the end of the task's original period.

# 4.3. Scheduling Both Periodic and Aperiodic Tasks

It is important to meet the regular deadlines of periodic tasks *and* the response time requirements of aperiodic events. ("Aperiodic tasks" are used to service such events.) Let us begin with a simple example.

Example 2: Suppose that we have two tasks. Let  $\tau_1$  be a periodic task with period 100 and execution time 99. Let  $\tau_2$  be an aperiodic task that appears once within a period of 100 but the arrival time is random. The execution time of task  $\tau_2$  is 1 unit. If we let the aperiodic task wait for the periodic task, then the average response time is about 50 units. The same can be said for a polling server, which provides 1 unit of service time in a period of 100. On the other hand, we can deposit 1 unit of service time in a "ticket box" every 100 units of time; when a new "ticket" is deposited, the unused old tickets, if any, are discarded. With this approach, no matter when the aperiodic event arrives during a period of 100, it will find there is a ticket for 1 unit of execution time at the ticket-box. That is,  $\tau_2$  can use the ticket to preempt  $\tau_1$  and execute immediately when the event occurs. In this case,  $\tau_2$ 's response time is precisely 1 unit and the deadlines of  $\tau_1$  are still guaranteed. This is the idea behind the deferrable server algorithm [3], which reduces aperiodic response time by a factor of about 50 in this example.

In reality, there can be many periodic tasks whose periods can be arbitrary. Furthermore, aperiodic arrivals can be very bursty, as for a Poisson process. However, the idea remains unchanged. We should allow the aperiodic tasks to preempt the periodic tasks subject to not causing their deadlines to be missed. It was shown in [3] that the deadlines of periodic tasks can be guaranteed provided that during a period of  $T_a$  units of time, there are no more than  $C_a$  units of time in which aperiodic tasks preempt periodic tasks. In addition, the total periodic and aperiodic utilization must be kept below  $U_a + ln[(2 + U_a)/(2U_a + 1)]$ , where  $U_a = C_a/T_a$ . And the server's period must observe the inequality " $T_a \le (T - C_a)$ ", where T is the period of a periodic task whose priority is next to the server.

Compared with background service, the deferrable server algorithm typically improves aperiodic response time by a factor between 2 and 10 [3]. Under the deferrable server algo-

rithm, both periodic and aperiodic task modules can be modified at will as long as the utilization bound is observed. Figure 4-1 illustrates the relative performance between background execution, the deferrable server algorithm, and polling. The workload is 60% periodic and 20% aperiodic. We assume a Poisson arrival process and exponentially distributed execution time for the aperiodic tasks. Since the mean aperiodic workload is fixed at 20%, short mean interarrival times imply short mean execution times. As we can see from Figure 4-1, the deferrable server is most effective for frequent arrivals with small service times.

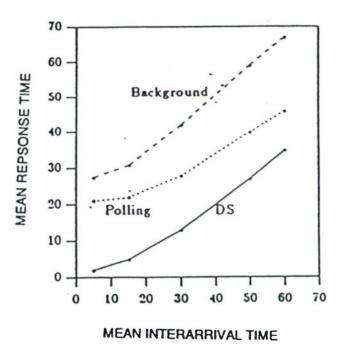


Figure 4-1: Scheduling Both Aperiodic and Periodic Tasks

A variation to the deferrable server algorithm is known as the *sporadic* server algorithm [13]. As with the deferrable server algorithm, we allocate  $C_a$  units of computation time within a period of  $T_a$  units of time. However, the  $C_a$  of the server's budget is not refreshed until the budget is consumed. From a capacity planning point of view, a sporadic server is equivalent to a periodic task that performs polling. That is, we can place sporadic servers at various priority levels and use only Theorems 1 and 2 to perform a schedulability analysis. Sporadic and deferrable servers have similar performance gains over polling, because any time an aperiodic task arrives, it can use the allocated budget immediately. When polling is used, however, an aperiodic arrival generally needs to wait for the next instant of polling. The sporadic server has the least runtime overhead. Both the polling and the deferrable servers have to be serviced periodically, even if there are no aperiodic arrivals. There is no

<sup>&</sup>lt;sup>7</sup>Early refreshing is also possible under certain conditions, see [13].

<sup>&</sup>lt;sup>8</sup>The ticket box must be refreshed at the end of each deferrable server's period.

overhead for the sporadic server until its execution budget has been consumed. In particular, there is no overhead if there are no aperiodic arrivals. Therefore, the sporadic server is especially suitable for handling emergency aperiodic events that occur rarely but must be responded to quickly.

From a scheduling point of view, polling, sporadic or deferrable server algorithms transform aperiodic activities into equivalent periodic activities that can be accommodated by the rate-monotonic scheduling algorithm. We can increase or decrease the capacity of a server task as if it were a normal periodic task.

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#### 5. Conclusions

In many real-time applications, neither the task set nor the task priorities can remain static throughout the mission. A change in operational mode often leads to the modification of task parameters as well as the addition of new tasks and deletion of old tasks. In this paper, we have developed a simple mode change protocol in a prioritized preemptive scheduling environment. We have shown that under this mode change protocol, there cannot be mutual deadlocks, and a high priority job can be blocked by lower priority jobs for at most the duration of one critical section, despite the addition and deletion of tasks during the mode change. We have shown that the worst-case mode change delay under this protocol is bounded and is generally much shorter than that possible in a commonly used cyclical executive.

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